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The intensity-dependent frequency shift in Thomson scattering from a thermal plasma

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Abstract. The intensity-dependent frequency shift in Thomson scattering, having been a controversial effect in the literature and being as yet unobserved, is calculated within classical electrodynamics for the interaction of a strong linearly polarised plane wave laser pulse with a high temperature thermal plasma. For the highest reported intensities an approximate redshift of $\Delta\omega/\omega_i = 10^{-2}$ is found. The simplifying assumptions involved in the derivation of the appropriate cross section are discussed. A few points relating to the possible detection of the frequency shift with available experimental facilities are touched upon.

1. Introduction

The non-linear interaction of free electrons with plane electromagnetic waves of high intensity has been extensively studied (Sarachik and Schappert 1970 and references therein). In the earlier stages of these investigations, the prediction of an intensity-dependent frequency shift in Thomson scattering (IDFS) by Sen Gupta (1952), Brown and Kibble (1964) and Gol'dman (1964) led to a controversy in the literature (Fried and Eberly 1964, Kibble 1965, Fried *et al* 1966, von Roos 1966, Stehle and de Baryshe 1966).

Although today there appears to be agreement on its existence, the IDFS is yet unobserved. In view of the ample experience with high intensity Thomson scattering in laser diagnostics of high temperature plasmas, together with recent progress towards higher available intensity (Kuroda *et al* 1976) and improved detecting devices (Pesin and Fabelinskii 1976), we investigate theoretically in this work the possibility of verifying the IDFS in a suitably adjusted experiment of this kind.

Accordingly, we evaluate in the following the pertinent scattering cross section within an idealised model that admits analytical calculations throughout. In § 2 we obtain the intensity modified scattering signal of a single electron with initial velocity $\boldsymbol{\beta}_0 = \boldsymbol{v}_0/c$. For vanishing initial velocity this quantity has been given by Sarachik and Schappert (1970). In § 3 we extend the result of § 2 to the intensity modified scattering cross section of a collection of electrons with a Maxwellian velocity distribution. For the special case of negligible intensity of the incident laser pulse, our cross section reduces to the well known expression used in Thomson scattering diagnostics of high temperature plasmas as evaluated by Sheffield (1972). In § 4 the modifications of this cross section under less idealised conditions than assumed in this work are discussed. We conclude with some remarks concerning the possible detection of the IDFS in Thomson scattering from a plasma with currently available experimental means.

2. Intensity modified single electron scattering signal

Our theory is tailored for a fully ionised thermal plasma of temperature of some 10^2 eV on which a very intense linearly polarised ultrashort Nd: glass laser pulse of some 5 ps duration is incident. This experimental situation justifies a fully classical treatment (Kibble 1968). Further, Gunn and Ostriker (1971) have shown that the effect of the inclusion of radiation reaction would essentially be the secular variation of otherwise constant quantities in the form of a factor $1+\mu^2\rho\omega_i\tau$, with $\rho=e^2\omega_i/mc^3$, $\mu=eE_0/m\omega_i c$ and τ, E_0, ω_i denoting the pulse duration, electric field strength and incident laser frequency respectively. The most intense pulses of a few picoseconds duration currently reported in the literature correspond to a value of μ^2 of approximately 10^{-2} (Kuroda *et al* 1976) so that we may safely neglect radiation reaction.

This permits us to base our investigations on standard results. In particular, we start with the solution of Krüger and Bovyn (1976) for the electron's velocity, which we reproduce here for later reference in our own notation:

$$\boldsymbol{\beta}=\frac{2b(\boldsymbol{a}-\boldsymbol{\mu}(\eta))+\boldsymbol{e}_i[1-b^2+(\boldsymbol{a}-\boldsymbol{\mu}(\eta))^2]}{1+b^2+(\boldsymbol{a}-\boldsymbol{\mu}(\eta))^2}. \tag{1}$$

In (1) the following abbreviations have been introduced: $\boldsymbol{\mu}(\eta)=\boldsymbol{e}_{\parallel}\mu(\eta)$, $\boldsymbol{a}=\boldsymbol{\beta}_{0\perp}(1-\beta_0^2)^{-1/2}$, $b=(1-\beta_{0i})(1-\beta_0^2)^{-1/2}$; $\boldsymbol{e}_i$ and $\boldsymbol{e}_{\parallel}$ are, respectively, the direction of propagation and polarisation of the laser pulse, $\beta_{0i}=\boldsymbol{\beta}_0\cdot\boldsymbol{e}_i$ and $\boldsymbol{\beta}_{0\perp}$ is the component of $\boldsymbol{\beta}_0$ perpendicular to $\boldsymbol{e}_i$.

This solution, however, hinges on the admissibility of a plane wave description of the incident pulse, which does not take account of intensity non-uniformities over the finite diameter of the laser pulse. Nevertheless, we are forced to resort to this approximation, since to date it has not been possible to deal with non-plane waves in the theory (Mitter 1975).

A specification of the function $\boldsymbol{\mu}(\eta)$ close to reality would be $\boldsymbol{\mu}=\boldsymbol{e}_{\parallel}\hat{\mu}\cos\eta\exp(-4\eta^2/\tau^2\omega_i^2)$ (Greenhow and Schmidt 1974, p 194). Instead, we make a simpler choice by assuming a rectangular pulse envelope:

$$\boldsymbol{\mu}(\eta)=\begin{cases}\boldsymbol{e}_{\parallel}\mu\cos\eta & |\eta|\leqslant 2N\\ 0 & |\eta|>2N,\end{cases} \tag{2}$$

with μ being related to the incident intensity $I_0=cE_0^2/8\pi$ by

$$\mu^2=8\pi c r_0^2 I_0/\omega_i^2 e^2 \tag{3a}$$

where r_0 is the 'classical electron radius'. N is related to the pulse duration by

$$\tau=4\pi N/\omega_i. \tag{3b}$$

This choice may seem to violate the requirement (Kibble 1965) that $\boldsymbol{\mu}(\eta)$ be a smooth pulse for the solution of Krüger and Bovyn (1976) to be valid. However, for the evaluation of the energy spectrum of the electron's scattering signal the particular pulse shape is not crucial. This is essentially a consequence of the Wiener–Khintchin theorem, which states that any function $\boldsymbol{\mu}(\eta)$ giving rise to the same autocorrelation of the scattered field leads to the same energy spectrum.

By substitution of (2) into (1) and integration with respect to time we obtain the quantity $\omega_i\boldsymbol{x}(\eta)/c$, where we drop the initial position $\boldsymbol{x}_0$, appearing in an integration constant, as only incoherent scattering will be considered in § 3. To calculate the scattering signal, we use the standard formula for the scattered energy per pulse, per unit frequency band and per unit solid angle as given by Jackson (1975, pp 670–71). By exploiting the partial periodicity, which the assumption (2) introduces into this expression, it can be cast, after an integration by parts, into the form

$$\frac{\mathrm{d}I(\omega)}{\mathrm{d}\Omega}=\frac{e^2\omega^2}{4\pi^2 c\omega_i^2}\frac{\sin^2(2\pi Nw\omega/\omega_i)}{\sin^2(\pi w\omega/\omega_i)}\left|\boldsymbol{s}\times\left(\boldsymbol{s}\times\int_0^{2\pi}\frac{\boldsymbol{\beta}}{1-\beta_i}\exp(\mathrm{i}F(\eta))\right)\right|^2. \quad (4)$$

Since the integrated term is down from the remaining integral by a factor of N^{-1}, with N from (3*b*) being of the order of 10^3, it has been neglected. The unit vector pointing from the scattering electron to the observer is denoted by $\boldsymbol{s}=(s_\parallel, s_\perp, s_i)$ with $s_\parallel=\boldsymbol{s}\cdot\boldsymbol{e}_\parallel$ and $s_i=\boldsymbol{s}\cdot\boldsymbol{e}_i$. The argument of the exponential is given by

$$F(\eta)=\frac{\omega}{\omega_i}\left[w\eta+\left(\frac{s_\parallel}{b}-\frac{a_\parallel(1-s_i)}{b^2}\right)\mu\sin\eta+\frac{(1-s_i)}{8b^2}\mu^2\sin 2\eta\right], \quad (5)$$

wherein w stands for

$$w=\frac{1-\boldsymbol{\beta}_0\cdot\boldsymbol{s}}{1-\beta_{0i}}+\frac{\mu^2}{2}\sin^2\left(\frac{\theta}{2}\right)\frac{1-\beta_0^2}{(1-\beta_{0i})^2}. \quad (6)$$

The expression (4) for the single electron scattering energy spectrum is a product of a rapidly varying 'diffraction' factor, reminiscent of the theory of the one-dimensional line grating, and a modulation factor, slowly varying with ω. The 'diffraction' factor possesses maxima of height $4N^2$ and width $\delta\omega\approx 2\pi/\tau$ at the frequencies $\omega=l\omega_i/w$, $l=1,2,3,\ldots$, reflecting the fact that (4), in principle, contains shifted harmonics besides the shifted incident frequency. For the special case of an electron initially at rest, these harmonics have been studied in detail by Sarachik and Schappert (1970).

The particular experimental situation that we have in mind allows some further simplifications. Firstly, for pulses as short as 5 ps, the width $\delta\omega$ of one 'diffraction' factor maximum will be approximately 7 Å, whereas the detector bandwidth will be 50 Å–100 Å (Sheffield 1975). Therefore, the exact shape of the maximum, which has already been altered by the introduction of a rectangular pulse, is irrelevant. Consequently, we replace the 'diffraction' factor by a sum of δ-functions:

$$\frac{\sin^2(2\pi Nw\omega/\omega_i)}{\sin^2(w\pi\omega/\omega_i)}\to\sum_{l=1}^{\infty}\frac{\tau\omega_i^2}{2\pi w}\delta(\omega-l\omega_i/w(\boldsymbol{\beta}_0)) \quad (7)$$

and the modulation factor by its value at $\omega=l\omega_i/w$. Secondly, a choice of the scattering plane perpendicular to $\boldsymbol{E}_0$ and the use of a plane polariser in the output oriented so as to accept scattered light polarised in the direction of $\boldsymbol{E}_0$ is advantageous experimentally (Sheffield 1972, 1975, Segre 1975). Hence we only need to keep the component of the vector integral (4) along $\boldsymbol{e}_\parallel$. With these simplifications, (4) reduces to

$$\frac{\mathrm{d}I(\omega)}{\mathrm{d}\Omega}=\frac{\tau e^2\omega^2}{8\pi^3 cw}\sum_{l=1}^{\infty}\delta(\omega-l\omega_i/w(\boldsymbol{\beta}_0))|I_l|^2 \quad (8)$$

wherein

$$I_l=b^{-1}\int_0^{2\pi}\mathrm{d}\eta\,(a_\parallel-\mu\cos\eta)\exp\left[\mathrm{i}l\left(\eta-\frac{2a_\parallel\nu\sin(\theta/2)}{b^2 w}\sin\eta+\frac{\nu^2}{4b^2 w}\sin 2\eta\right)\right] \quad (9)$$

with $\boldsymbol{s}\cdot\boldsymbol{e}_i=\cos\theta$ and $\nu=\mu\sin(\theta/2)$.

Since in the experimental situation considered, both μ^2 and the components of $\boldsymbol{\beta}_0$ are of the order of 10^{-2}, it suffices to expand (9) in powers of these quantities. It turns out that the I_l for $l \geqslant 2$ are smaller than correction terms that have already been neglected, so that in (8) only the term $l = 1$ survives:

$$|I_1|^2 \approx \mu^2 \pi^2 (1 + 2\beta_{0i} - \tfrac{1}{4}\nu^2). \tag{10}$$

If we now introduce as usual the differential scattering cross section $d\sigma(\omega)/d\Omega$ by dividing (8) by the incident pulse energy per unit area, $I_0\tau$, we find with (3a) for the single electron scattering signal

$$\frac{d\sigma(\omega)}{d\Omega} = r_0^2 \left(\frac{\omega}{\omega_i}\right)^3 \left(1 + 2\beta_{0i} - \frac{\nu^2}{4}\right) \delta\left(\omega - \frac{\omega_i}{w(\boldsymbol{\beta}_0)}\right). \tag{11}$$

3. Intensity modified incoherent spectrum from a Maxwellian ensemble of electrons

In the experimental situation considered, with an electron density $n_e \approx 10^{12}$ cm^{-3} and a scattering angle $\theta = \pi/2$, the condition $\alpha = [\lambda_D(\omega_i/c)\sin(\theta/2)]^{-1} \ll 1$ is satisfied and the total scattering signal is a sum of contributions of the form (11) (Segre 1975). Assuming a Maxwellian velocity distribution, the differential cross section is given by

$$\frac{d\sigma(\omega)}{d\Omega} = r_0^2 \frac{n_e}{\pi^{3/2}} \left(\frac{c}{a}\right)^3 \left(\frac{\omega}{\omega_i}\right)^3 \int d^3\beta_0 \left(1 + 2\beta_{0i} - \frac{\nu^2}{4}\right) \delta\left(\omega - \frac{\omega_i}{w(\boldsymbol{\beta}_0)}\right) \exp\left(-\frac{\beta_0^2 c^2}{a^2}\right), \tag{12}$$

with $a = (2kT/m)^{1/2}$.

The integration is straightforward if again only lowest order correction terms in $\boldsymbol{\beta}_0$ and μ^2 are retained and yields, after the introduction of $\Delta\omega = \omega - \omega_i$:

$$\frac{d\sigma(\omega)}{d\Omega} = \frac{n_e r_0^2 q}{\pi^{1/2}\omega_i}\left(1 + \frac{\Delta\omega}{\omega_i} - \frac{3\nu^2}{4}\right) \exp\left\{-q^2\left[\left(\frac{\Delta\omega}{\omega_i} + \frac{\nu^2}{2}\right)^2 \left(1 - \frac{\Delta\omega}{\omega_i} - \frac{\nu^2}{2}\right) + 2\frac{\Delta\omega}{\omega_i}\frac{\nu^2}{2}\left(\frac{\Delta\omega}{\omega_i} + \frac{\nu^2}{2}\right)\right]\right\}, \tag{13}$$

where $q = c/2a \sin(\theta/2)$.

Finite transit time corrections have not been included in (13), but can easily be incorporated according to the rules given by Segre (1975). Figure 1 gives a graph of (13) for $T_e = 100$ eV, $\theta = \pi/2$ and $\mu^2 = 0$, 4×10^{-3} and 4×10^{-2} respectively.

Before we proceed to discuss figure 1, note that (12) can be cast into a form more appropriate for the following discussion by introducing the average velocity $\boldsymbol{\beta}_{av}$ instead of $\boldsymbol{\beta}_0$ as the variable of integration. $\boldsymbol{\beta}_{av}$ is defined as the time average of (1):

$$\boldsymbol{\beta}_{av} = \left(\boldsymbol{\beta}_0 + \boldsymbol{e}_i \frac{\mu^2}{4} \frac{1 - \beta_0^2}{1 - \beta_{0i}}\right)\left(1 + \frac{\mu^2}{4} \frac{1 - \beta_0^2}{1 - \beta_{0i}}\right)^{-1}. \tag{14}$$

This can be inverted to give

$$\boldsymbol{\beta}_0 = \left(\boldsymbol{\beta}_{av} - \boldsymbol{e}_i \frac{\mu^2/4}{1 + \mu^2/2} \frac{1 - \beta_{av}^2}{1 - \beta_{av,i}}\right)\left(1 - \frac{\mu^2/4}{1 + \mu^2/2} \frac{1 - \beta_{av}^2}{1 - \beta_{av,i}}\right)^{-1}. \tag{15}$$

Substituting (15) into (12), $w(\boldsymbol{\beta}_0)$ in the argument of the δ-function as given by (6) is transformed into $(1 - \beta_{av,s})/(1 - \beta_{av,i})$. This means that in a scattering process of the above kind, admitting the concept of average velocity $\boldsymbol{\beta}_{av}$, we can obtain a first

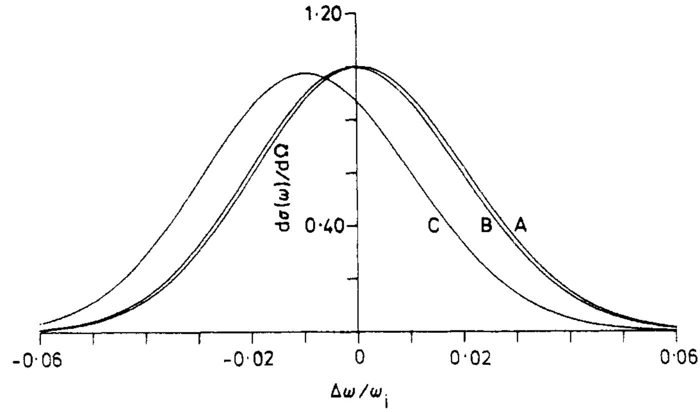

Figure 1. The intensity modified differential scattering cross section $d\sigma(\omega)/d\Omega$ as a function of $\Delta\omega/\omega_i = (\omega - \omega_i)/\omega_i$ for $T_e = 100$ eV, $\theta = \pi/2$ and $\mu = 0$, 4×10^{-3} and 4×10^{-2} for curves A, B and C respectively. The units on the axis of ordinates are arbitrary.

approximation to the scattering cross section by finding $\boldsymbol{\beta}_{av}$ for each electron and by subsequently evaluating the integral

$$\frac{d\sigma(\omega)}{d\Omega} = \frac{n_e r_0^2 c^3 \omega^3}{\pi^{3/2} a^3 \omega_i^3 \bar{\mu}^2} V^{-1} \int d^3x_0 \int d^3\beta_{av} \mu_{av}^2(\boldsymbol{x}_0, \boldsymbol{\beta}_0) \delta\left(\omega - \omega_i \frac{1-\beta_{av,i}}{1-\beta_{av,s}}\right) \exp\left(-\frac{c^2\beta_0^2}{a^2}\right) \tag{16}$$

to the appropriate order. In μ_{av}^2, being the average intensity felt by the electron along its trajectory, and in the exponential, $\boldsymbol{\beta}_0$ is to be expressed by $\boldsymbol{\beta}_{av}$. $\boldsymbol{x}_0$ is the position of the electron at some initial reference time, $\bar{\mu}^2$ is the mean intensity and V is a suitable volume. In fact, one can easily show that by substituting (15) into (16), one recovers the scattering cross section (13) to lowest order.

4. Discussion

If ν is set equal to zero in (13), i.e. if the influence of the intensity of the laser pulse on the average motion of the electrons is ignored, then (13) reduces to the well known result of laser diagnostics of high temperature plasmas as given, e.g., by Segre (1975). Consequently, curve A in figure 1, corresponding to $\mu = 0$, exhibits the familiar relativistic blueshift, which, despite its smallness in the case of $T_e = 100$ eV was measured by Gondhalekar and Kronast (1973) in the incoherently scattered 0·69 μm light from a theta pinch plasma. From curve B, which corresponds to $\mu^2 = 4\times10^{-3}$, it is seen that the laser-induced uniform drift velocity of all electrons just overcompensates the relativistic blueshift, so that the centre of the line is slightly redshifted. Finally, for $\mu^2 = 4\times10^{-2}$, or in customary units, $I_0 = 10^{17}$ W cm^{-2}, which corresponds to the intensity achieved by the laser system described by Kuroda *et al* (1976), curve C exhibits a redshift almost an order of magnitude larger than the blueshift in case A.

In conclusion, we would like to mention a number of effects which would modify the idealised cross section (13) in an actual experiment. First, the uniform average velocity (14), acquired by all electrons regardless of their 'initial' position $\boldsymbol{x}_0$, is a consequence of our assumption of a plane wave. Schmidt and Wilcox (1973) have demonstrated that the electron quite generally suffers a net acceleration in the direction of the negative intensity gradient of the wave. Hence, in the spatial intensity distribution of a real laser pulse, different electrons will acquire different average velocities, depending on the individual trajectories through different pulse portions. The direction and magnitude of $\boldsymbol{\beta}_{\mathrm{av}}$ will depend on the average negative intensity gradient and on the average intensity μ_{av}^2 experienced by the electron along its path. For a given spatial intensity distribution, an accurate evaluation of the scattering cross section would have to start with the work of Schmidt and Wilcox (1973). Here we are only concerned with a crude estimate of the effects of intensity non-uniformity that can be based on (16). For this it suffices to approximate $\boldsymbol{\beta}_{\mathrm{av}}$ for an axisymmetric pulse by

$$\boldsymbol{\beta}_{\mathrm{av}} = \boldsymbol{\beta}_0 + \beta_{\mathrm{D,r}}(r)\boldsymbol{e}_{\mathrm{r}} + \beta_{\mathrm{D,l}}(r)\boldsymbol{e}_{\mathrm{l}}, \tag{17}$$

where the subscripts D, r and D, l stand for radial and longitudinal drift velocity respectively. $\boldsymbol{e}_{\mathrm{r}}$ is the unit vector pointing away from the pulse axis. The total $\boldsymbol{\beta}_0$-dependence of $\boldsymbol{\beta}_{\mathrm{av}}$ is taken to be represented by the first term on the right of (17), so that $\beta_{\mathrm{D,r}}$ and $\beta_{\mathrm{D,l}}$ are functions of the electron's initial distance r from the pulse axis only. This requires the quantity $I_0\ /|(\partial I_0/\partial r)|$ to be large compared with $\beta_0 c\tau$. Substituting (17) into (16) and performing the integrations with respect to β_{av} and the azimuthal angle, one finds that upon the inclusion of a spatially slowly varying intensity distribution the scattering cross section becomes a weighted sum of contributions of the general form (13). A typical contribution is thereby characterised by a width approximately increased by $2\beta_{\mathrm{D,r}}$ as compared with (13) and by an approximate redshift of $\beta_{\mathrm{D,l}}$. We thus obtain the plausible result that $\boldsymbol{\beta}_{\mathrm{av}}\cdot\boldsymbol{e}_{\mathrm{r}}$ tends to increase the linewidth while $\boldsymbol{\beta}_{\mathrm{av}}\cdot\boldsymbol{e}_{\mathrm{l}}$ leads to a redshift as before. Since the redshift in (13) is based on an overall value $\mu = \bar{\mu}$, we conclude that in a more accurate calculation the electrons interacting with the central region of the pulse, where $\mu > \bar{\mu}$, would contribute a redshift larger than, and a width close to, that in (13), while those interacting with the peripheral region, where $\mu < \bar{\mu}$, would contribute a redshift smaller than, but a width larger than, that in (13). Consequently, we can expect the resulting redshift to be close to the 'average' redshift as exhibited by (13). However, the resulting linewidth would not longer be directly related to the plasma temperature as it is in ordinary plasma diagnostics.

The influence of a magnetic field, necessary to confine a 100 eV plasma, need not be included in the derivation of (13), because the spectrum cannot exhibit magnetic modulation as, e.g., in the experiment of Evans and Carolan (1970), since even in a field of 10 kG the electron gyroperiod would be an order of magnitude larger than the assumed pulse duration of 5 ps.

So far, we have only dealt with the single particle behaviour of the plasma. In the interaction with such intensities as 10^{17} W cm^{-2}, however, its collective behaviour will become important and a multitude of parametric instabilities will develop (Drake *et al* 1974). The assumptions underlying our calculations are therefore only valid as long as parametric instabilities have not appreciably developed. Taking the inverse of the electron plasma frequency as a crude measure for the time scale on which they develop (Sheffield 1975), we obtain for an electron density of $n = 10^{12}$ cm^{-3} a characteristic time of $\omega_{\mathrm{pe}}^{-1} \approx 20$ ps. That is why we based our calculation on the shortest available high intensity pulses of 5 ps (Kuroda *et al* 1976), whose duration places an upper bound on the admissible electron density.

It is clear that both ultrashort pulse duration and low plasma density involve severe problems as regards an experimental detection of the IDFS. On the other hand, the recent progress in optoelectronics as reviewed by Pesin and Fabelinskii (1976) is quite remarkable. Thus, to deal with the pulse duration of a few picoseconds one could employ an ultrafast optical shutter, e.g. a rapidly bleachable dye, triggered by a portion of the primary pulse so as to transmit only the scattering signal but no continuum radiation emitted by the plasma before and afterwards. It is true that the necessary density of $n_e \leqslant 10^{12}\ cm^{-3}$ excludes the use of photocathodes as detecting devices because of their very low quantum efficiency at $1{\cdot}06\ \mu m$. But they could be replaced by photodiodes specifically designed for this wavelength (e.g., the YAG-series by EG and G Inc. Electro-Optics Division, Salem, Mass., USA).

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